

## Ch. 3: Correlation & Regression

- Exploring relationships between 2 variables
- Scatterplots
- Linear Regression
- Exercise 03
- Correlation
- Other Correlation Coefficients

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## Bivariate relationships

- “is factor A related to factor B”?
- Methods of analysis:
  - Anecdotal / Clinical -- often forms the basis for further systematic research & data collection
  - Numerically -- check values & % at extremes
  - Visually -- scatterplots
    - easy to see relationships and problems w/ data
    - hard to prove / test
  - Statistically -- correlation & regression
    - hard to detect problems w/data
    - easy to test hypothesis

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## Anecdotal / Clinical

- Many interesting findings in psychology first originate from non-scientific approaches
- “Intuition” that something is related through experiencing multiple situations
- Pattern recognition
- Human brains are both excellent and terrible pattern recognizers
- Problems -- faulty memory, confirmation biases, prejudice, etc...
- First step after a “gut” feeling is to begin collecting data.

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## Simple numerical analysis

- Simplify the situation by using Categorical variables (or reducing Continuous variables to Categorical variables)
- Use extreme cases to maximize effect
- Compute percentages in a 2x2 matrix
- Do the results suggest an effect?
- Compute Chi-square statistic to judge significance

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## Example

- “I think there is brain dysfunction in HIV disease” as measured by neuropsychological testing
- Medical status: control vs. HIV+ symptomatic
- NP test results: normal vs. impaired

		Medical Status	
		Control	HIV+
NP Status	Normal	85%	52%
	Impaired	15%	48%

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## Issues

- Pro: Results are very easy to understand from a human or clinical point of view.
- Con: dividing continuous variables into two values throws away a lot of data and statistical power
- Graphical and Statistical methods should be used as well.

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# Scatterplots

- Graph two variables in relation to each other on two-dimensional X, Y axis
- Easy to see relations between data
- Easy to see problems with data
- Hard to prove or determine if an apparent relationship is “significant”
- Hard to interpret data clinically or in “common sense” terms

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# Scatterplots

x	y
10	8.04
8	7.58
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68



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# Linear Regression

- Assume that two variables are related, and that this relationship is linear -- model the data by a simple straight line for the data.
- For any given data set, we pick the line that best “fits” our data
- Similar terms: linear regression, fitting a line, finding the trend, creating a trendline, best fit line, etc.
- Residuals = difference between prediction and actual value
- Linear Regression minimizes the square of the residuals, often called “Ordinary Least Squares”

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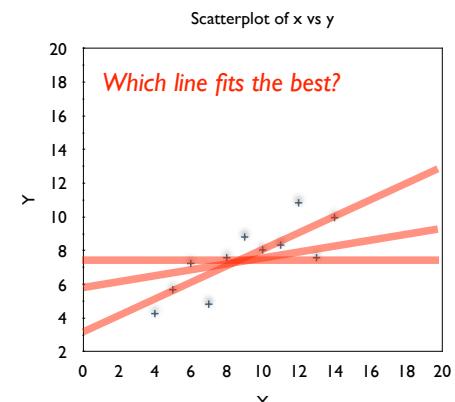
# Linear Regression

**Equation:**  

$$y = 3.0 + 0.5x$$

**Correlation**  

$$r_{x,y} = 0.816$$

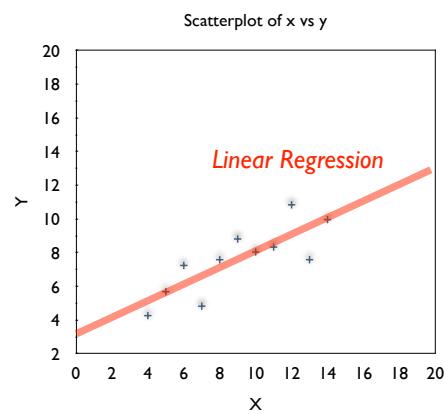


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# Anscombe's Quartet I

x	y
10	8.04
8	7.58
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68

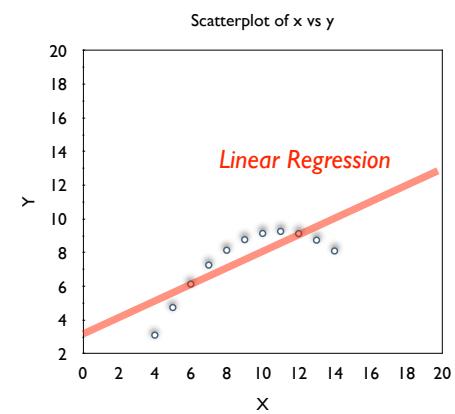


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# Anscombe's Quartet II

x	y
10	9.14
8	8.14
13	8.74
9	8.77
11	9.26
14	8.1
6	6.13
4	3.1
12	9.13
7	7.26
5	4.74

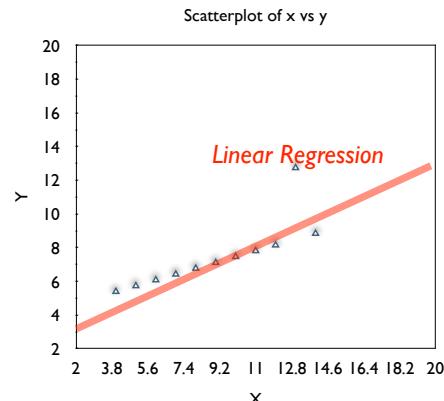


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## Anscombe's Quartet III

X	Y
10	7.46
8	6.77
13	12.74
9	7.11
11	7.81
14	8.84
6	6.08
4	5.39
12	8.15
7	6.42
5	5.73

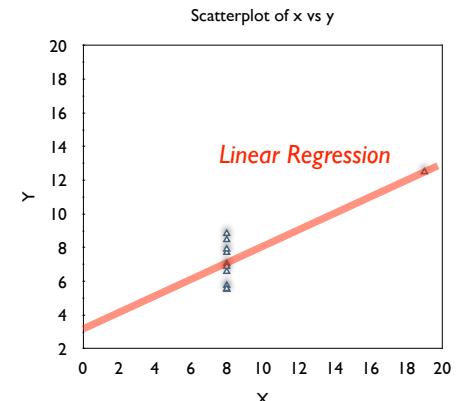


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## Anscombe's Quartet IV

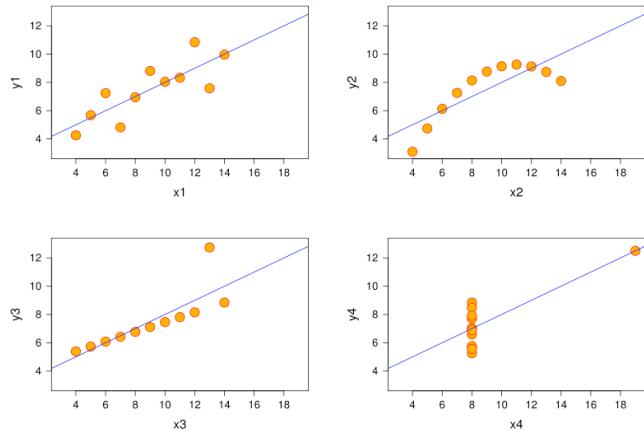
X	Y
8	6.58
8	5.76
8	7.71
8	8.84
8	8.47
8	7.04
8	5.52
19	12.5
8	5.56
8	7.91
8	6.89



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## Anscombe's Series 1-4



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## Anscombe's Quartet Summary

- The 4 series of data, though very different, have identical linear regression equations and identical correlations
- Each series has a Quantitative correlation, but it's clear (visually) that the relationships are Qualitatively different
- Each series should probably be handled differently, through techniques such as:
  - Trimmed Least Squares
  - Robust regression
  - Graph Your Data!

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## Linear Regression Equation

$$Y' = a + bX$$

$$Y' = \text{predicted } Y$$

$$X = \text{actual } X$$

$$b = \text{slope}$$

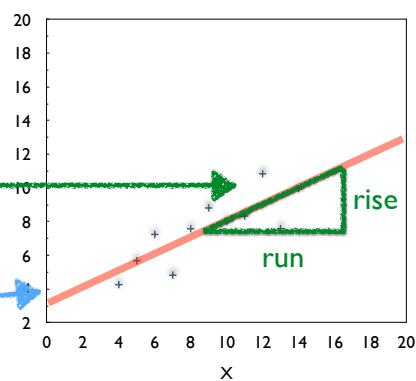
$$DY/DX$$

$$(\text{rise over run})$$

$$a = \text{intercept}$$

$$Y \text{ value when } X = 0$$

Scatterplot of x vs y



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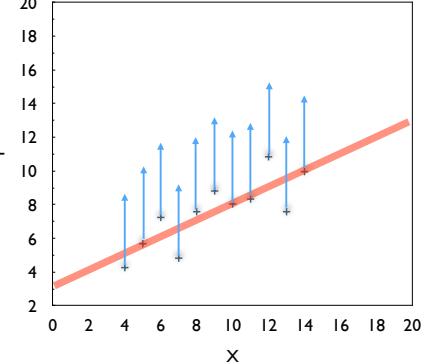
## Adjusting Intercept

$$Y' = a + bX$$

$$a = \text{intercept}$$

Intercept can be raised by adding same amount to all Ys

Scatterplot of x vs y



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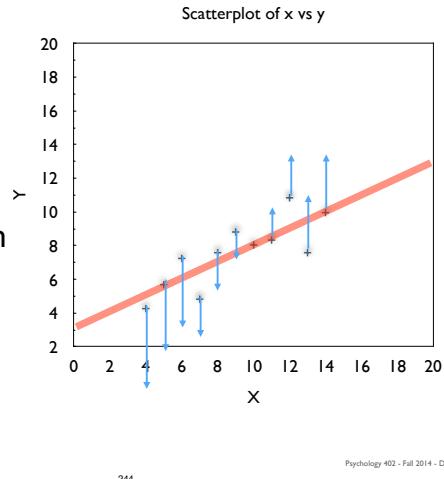
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## Adjusting Slope

$$Y' = a + bX$$

$b$  = slope

Slope change by adding more to right side, subtracting from left side



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## Residuals in Linear Regression

- $X$  : independent variable
- $Y$  : dependent variable
- Model: predict  $Y$  from  $X$
- $Y'$  : (Y prime) = predicted  $Y$
- $Y' = a + bX$
- Prediction is (usually) incorrect. Difference between predicted ( $Y'$ ) and actual ( $Y$ ) is called a “Residual” =  $(Y - Y')$
- Calculation of best fit line minimizes the sum of the squared residuals  $\Sigma(Y - Y')^2$

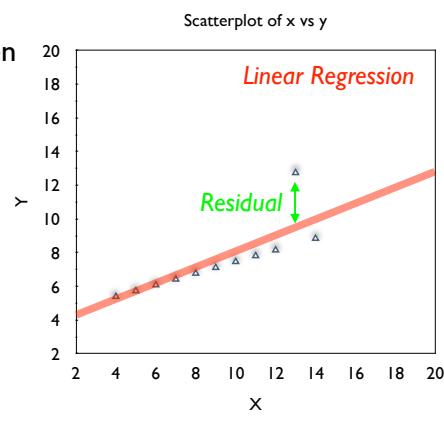
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## Residuals in Linear Regression

Residual is difference between actual  $Y$  and predicted  $Y'$  ( $Y - Y'$ )

Graphically it is equal to how far away (vertically) a point is from the linear regression line

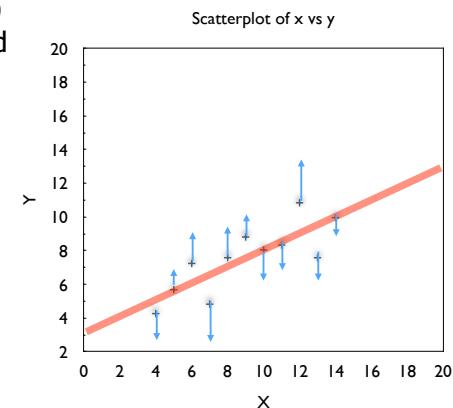


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## Adjusting Residuals

Residuals (error) can be increased by moving  $Y$  values further away from the regression line.

If above line, move higher.  
If below line, move lower.



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## Residuals, Variance, $R^2$

- Residual =  $(Y - Y')$
- Squared residual =  $(Y - Y')^2$
- Sum of squared residuals =  $\Sigma(Y - Y')^2$ 
  - Linear regression minimizes this value
- SSR is hard to interpret
- $R^2$ 
  - $R^2 = 1 - (SSR/SST)$
  - Coefficient of Determination
  - Unexplained Variance
  - Ranges from 0 to 1 (often 0% to 100%)

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## Standard Error of Estimate

- Residual =  $(Y - Y')$
- Standard Deviation of residuals
  - measure of “average” error
  - aka “Standard Error of Estimate”

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# Correlation (r) Pearson's r

- Pearson's Product-Moment Correlation
- Measures the strength of the linear relationship between two variables
- Ranges between -1.0 and +1.0
- Is a special case of linear regression, when both X and Y have been turned into Z scores.
- $r$  is transitive (correlation between X and Y is same as correlation between Y and X)
- $R^2$  = “explained variance” is the proportion of variation in the data explained by the model.
- $R^2$  ranges from 0 to 1.0 (0% to 100%)

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# Regression vs. Correlation

	Linear Regression	Correlation
<b>Scores</b>	Raw	$Z$
<b>Mean, Std Dev</b>	sample means sample Std Dev	0 1
<b>Equation</b>	$Y' = a + bX$	$Y' = rX$
<b>Slope</b>	$b = \text{change in } Y \text{ per change in } X$	$r = \text{correlation coefficient}$
<b>Slope</b>	meaningless	$R$
<b>Transitive?</b>	no	yes, $R$

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# Other Correlation Coefficients

- Continuous (interval & ratio): Pearson's  $r$
- Ordinal (Ranked): A B C D... 1st, 2nd, 3rd...
  - Spearman's Rho: correlation between two ordinal / ranked variables.
- Dichotomous (yes/no, one/zero, T/F, Male/Female, Pass/Fail...)
  - True vs. Artificial?

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# Continuous vs. Dichotomous

Type of X / Type of Y	Continuous	Artificial Dichotomous	True Dichotomous
<b>Continuous</b>	Pearson $r$	Biserial $r$	Point biserial $r$
<b>Artificial Dichotomous</b>	Biserial $r$	Tetrachoric $r$	Phi
<b>True Dichotomous</b>	Point biserial $r$	Phi	Phi

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# Correlation : Issues

- Technical / Calculation :
  - Non-normal distribution
  - Non-linear data and relationships
  - Outliers, data errors
  - Restricted Range
  - Shrinkage
- Interpretation:
  - Correlation =? Causation
  - Third variable explanations

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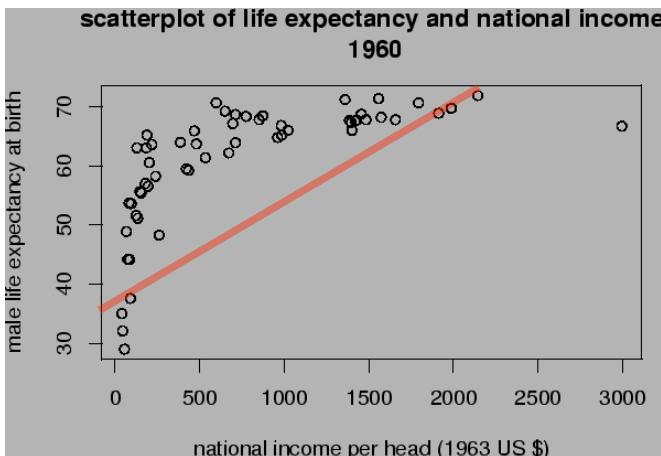
# Non-linearity

- Linear regression & Correlation assume a linear relationship between X and Y
- Are most real-world relationships linear?
- Examples of non-linearity
- Solutions:
  - Intentionally restrict range of X
  - Rank variables then use Spearman's Rho
  - Transform variables (log, root, square, cube, etc.)
  - Use higher-order (polynomial) curve fitting, such as  $Y = a + bX + cX^2 + dX^3 \dots$

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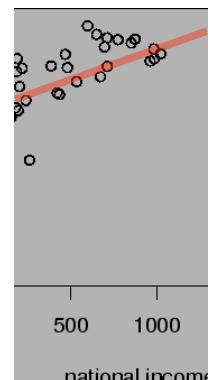
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## Life expectancy / national income



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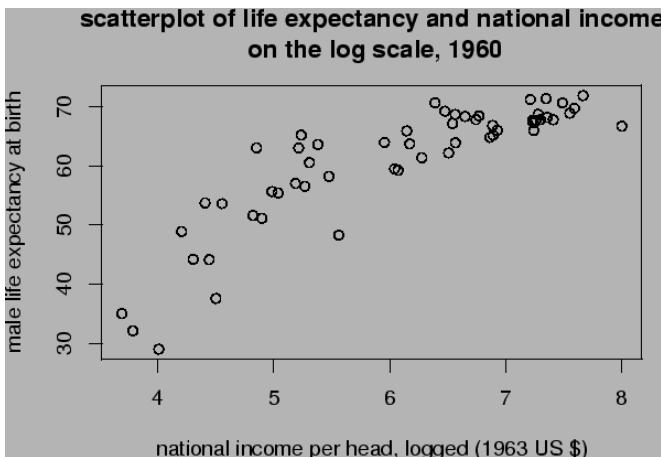
## restrict range of X



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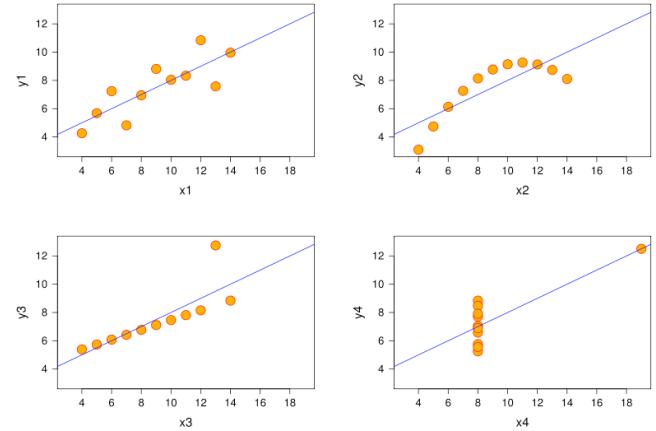
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## log transform X (or Y)



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## Outliers & Data Errors?



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## Correlation = Causation?

- A relationship (linear or otherwise) between X and Y tells us nothing about whether X causes Y
- Lack of correlation between X and Y does not mean that X doesn't cause Y
- Sleeping with your shoes on is correlated with waking up with a headache
- Ice cream sales are positively related to increase in drowning

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## Shrinkage

- Least-squares regression attempts to fit the data set presented to it by reducing the observed residuals.
- This data set contains random errors.
- Thus, the parameters (equations) estimated for the linear regression line (and correlation coefficient) and residuals usually be higher than would be found in a separate data set.
- This reduction is called “Shrinkage”
- Cross-validation is best way to deal with it

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## Cross Validation

- Step 1: With a given data set, compute the linear regression line that fits this data.
- Step 2: Apply this linear regression equation to a different data set.
- Step 3: Calculate the observed error in step 2. This is typically higher than seen in step 1, and a much better measure of fit.
- Note: sometimes you may artificially “create” two data sets by splitting a single data set in half.

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## Hypothesis Testing

- All parameters (equations) we estimate from data have inherent error
- How do we know if a given estimate is correct?
- How big is the error likely to be (confidence intervals)?
- Inferential Statistics - covered later
  - Formulas to calculate probability, confidence intervals.
  - Higher N is better
  - “statistical significance” not the same as “clinical significance”

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## Statistical and Clinical Significance

- These two terms are often confused and have very different meanings
- Statistical Significance: changes in DV are very unlikely to have been the result of random effects or chance. Often expressed as a P value ( $p < .01$ , or less than 1% chance to see these effects under  $H_0$ )
- Clinical Significance : changes in DV are large enough to matter; the change was not trivial. If we accept  $H_1$ , the conclusion is that  $H_1$ ’s effect size is important.
  - depends on context. often evaluated in terms of cost/benefit or risk/benefit tradeoff

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## Significance

- Example 1:
  - Two dice, Roll each once
  - Results: get a 3 and a 5
- Example B:
  - Two dice, Roll each 100 times
  - Results: Die A = 3.0, Die B = 3.10
- Example C:
  - Two dice, Roll each 100 times
  - Results: Die A = 3.0, Die B = 5.0

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## Lies, damned lies, and statistics

- Statistical significance (P) is a function of...
  - Errors of measurement (E)
  - Effect Size (D)
  - Sample Size (N)
- $p \sim E / (D \times N)$

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## Reporting Results

- “Men had higher IQ than women. Results were statistically significant  $p < .001$ ”
- Effect Size (D)
- P-value
- Probability of Type I error ( $\alpha$ )
- Probability of Type II error ( $\beta$ )

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