Ch. 2 - Measurement & Stats

Review

- Themes
  - 18th-19th century
  - 19th-20th century
- Theories of Human Development
  - Creationism
  - Polygenism
  - Evolution
  - Genetics
- Controversy
  - IQ testing of various groups

Measurement & Stats

- Why numbers?
- Distribution & Graphs : Histogram
- Central Tendency
- Mean, SoR, SSR, Variance, Standard Deviation
- In-class exercise
- Population vs. Sample
- Measurement Scales
- Precision vs. Accuracy
- Logic and Logical Fallacies Descriptive vs. Inferential Statistics
- Norms

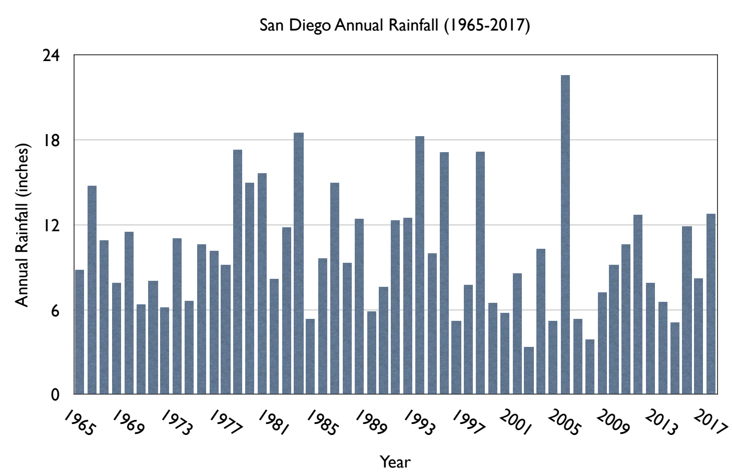
Basic Statistics

- Why use numbers?
  - Pros:
    - convenient, succinct
    - universal
    - well-defined, repeatable
  - Cons:
    - precision vs. accuracy
    - numerical fallacy

Tabular Data

Year	Rainfall (inches)
1965	8.81
1966	14.76
1967	10.86
1968	7.86
1969	11.48

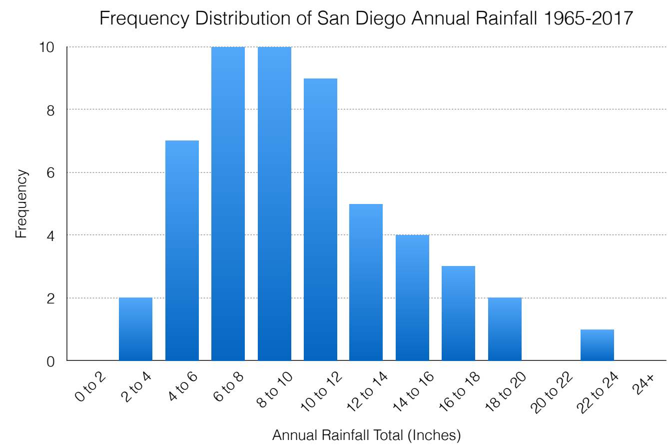
Data Distributions



Histogram

- Frequency Distribution
- Invented by Karl Pearson
- Shows data from *one* variable only
- Data is collected into groups "bins"

Histogram



Ranks, Percentiles

- Given a distribution of scores, and a single score
- **Rank** = the item # of the single score when sorted high to low
- **Percentile Rank** = the % of scores which are lower than the given score
- **Percentile** = the score at which a given percent of scores are below a given score

- Note: "**Percentile**" often used informally to mean "**Percentile Rank**"

Rank & Percentile

- Infant mortality per 1000 live berths

- Sorted low to high

Country	Score
Sweden	2.4
Japan	3.4
France	4.5
USA	7.5
Colombia	20.4
China	37.9
Bolivia	66.4
Ethiopia	142.6
Mozambique	148.6
Zambia	168.1

Rank & Percentile

- Determine Rank #

Country	Score	Rank
Sweden	2.4	1
Japan	3.4	2
France	4.5	3
USA	7.5	4
Colombia	20.4	5
China	37.9	6
Bolivia	66.4	7
Ethiopia	142.6	8
Mozambique	148.6	9
Zambia	168.1	10

Percentile Rank

Percentile Rank = # of cases with worse value divided by # of cases
e.g. France is third of 10 (it has 7 cases with worse values)
7 / 10 = 70%

Country	Score	Rank	%ile Rank
Sweden	2.4	1	90
Japan	3.4	2	80
France	4.5	3	70
USA	7.5	4	60
Colombia	20.4	5	50
China	37.9	6	40
Bolivia	66.4	7	30
Ethiopia	142.6	8	20
Mozambique	148.6	9	10
Zambia	168.1	10	0

Describing Distributions

- Why do this? Large sets of numbers are hard to work with. Easier to reduce dozens, hundreds or thousands of data points down to a few numbers.
- Issue: Any time you reduce the number of data (called “Degrees of freedom” you are throwing away data).
- In essence we are Modeling our data using a simplification.
- “All models are wrong, some models are useful”

Descriptive stats. problems

- Descriptive statistics rely on certain assumptions. When those assumptions are not met, weird things happen.
- Example: Joe Smith is 6 feet tall, his child is 1 foot tall. Thus, the average height in the Smith household is 3.5 feet.
- If you are sitting in bar, and Bill Gates walks in, suddenly you are (on average), a multi-millionaire.

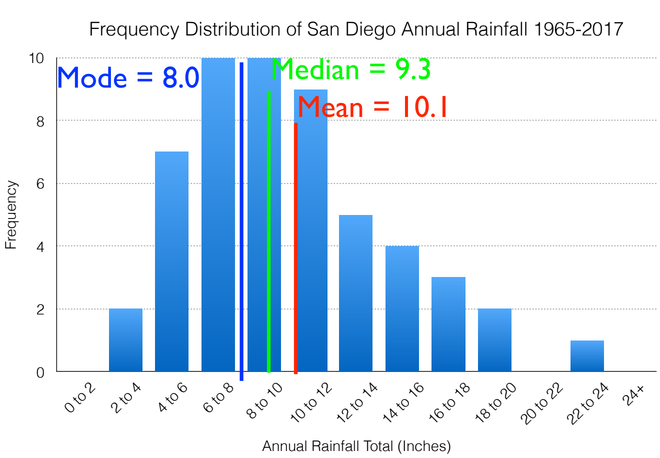
Notation

- Different ways of writing the same thing
- Square Root:
  - sqrt(X) $\sqrt{X}$
- Squared:
  - X^2
  - X**2 X^2

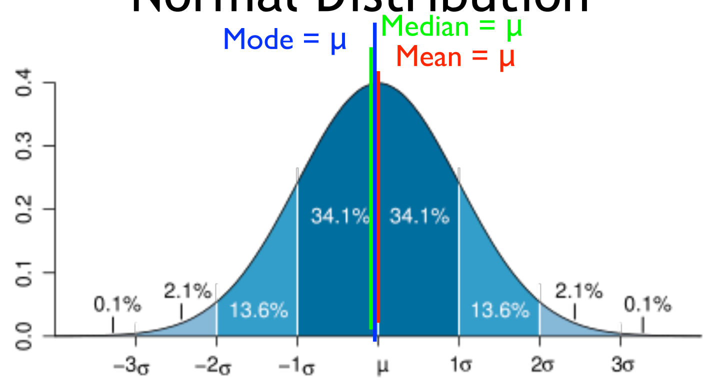
Central Tendencies

- Assuming the values tend to cluster around a point, what is the point?
- **Mean** : most common statistic, commonly referred to as the “average”. Formula ∑X / N
- **Mode**: the most common value in a set
  - rare to use in statistics
- **Median**: the middle-most value in a set
  - the value at which half are above and half are below. Aka the 50th percentile.

Histogram

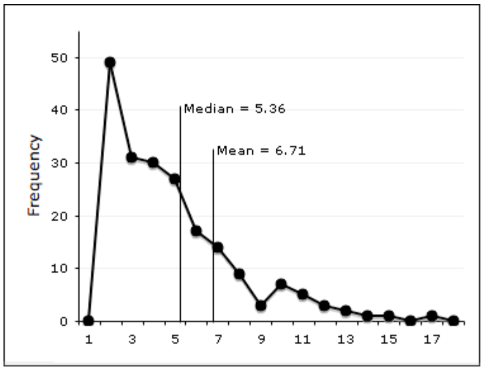


Normal Distribution



In a normal distribution, the mean, mode, and median are all the same

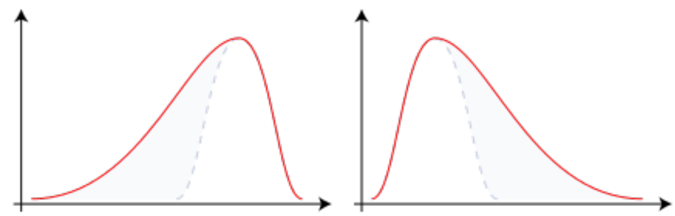
Skewed Distribution



In a skewed distribution, the mean, mode, and median are all often different

Skew

- negative skew : fatter tail on the left
- positive skew : fatter tail on the right



Negative Skew
"Left Skew"

Positive Skew
"Right Skew"

Measures of Central Tendency 1

	Description	Algorithm	Formula
Mean	the "average"	sum values, divide by N	$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$
Median	the "middle-most value"	sort values, find middle value	50th percentile
Mode	the "most common" value	find most frequent value	…

Measures of Central Tendency 2

Behavior:	Normal Distribution	Skewed Distribution
Mean	same	overly affected by outliers
Median	same	fairly resistant to outliers
Mode	same	resistant to outliers

Measures of Dispersion 1

- How do individual values vary compared to the average value?
- "for a typical value, how far away is it from the mean"
- "Difference score" or "residual" can be calculated as the difference between the actual score and the mean. In other words, $d_i = x_i - \bar{X}$
- Take the average (mean) of the difference scores.
- Average difference score = Sum(d) / N

Average Difference Score

	Score (x)	Mean ($\bar{X}$)	Difference d = (x - $\bar{X}$)
	2	6	-4
	3	6	-3
	9	6	3
	11	6	5
	14	6	8
	1	6	-5
	6	6	0
	4	6	-2
	5	6	-1
	5	6	-1
Sum	**60**	**60**	**0**
Mean	**6**	**6**	**0**

Sum of Residuals

- Given N samples of x : $x_1, x_2, x_3 \dots x_N$
- mean of x $$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$
- residuals $$d_i = x_i - \bar{x}$$
- Sum of Residuals

$$\sum_{i=1}^{N} d_i = 0$$

Sum of Residuals

- The "average difference score" score will *always* equal zero
- Solution:
  - Square the residuals before adding: removes the negative values.
  - "SSR" or Sum of Squared Residuals
- Problem: units are hard to interpret
  - is SSR = 0.000000342 high or low?
  - is SSR = 2343153249 high or low?

SSR

- Given N samples of X: $x_1, x_2, x_3 \dots x_N$
- mean of x $$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$
- residuals $$d_i = x_i - \bar{x}$$
- Sum of Squared Residuals (SSR)

$$SSR = \sum_{i=1}^{N} (d_i)^2$$

Variance

- SSR is hard to interpret - it depends on N
- Take the average to remove the influence of N
- The average of the squared residuals is called Variance (S^2)
- Problem: units are still squared.
  - if you were measuring height in meters, your variance is now in meters^2

Variance

- Variance = SSR/N
- Variance = mean of squared residuals

$$S^2 = \frac{\sum_{i=1}^{N} (d_i)^2}{N}$$

$$S^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}$$

Standard Deviation

- Improving on Variance:
- The square root of Variance (S^2) gives S, which is called "Standard Deviation".
- Also abbreviated SD, StdDev or σ (Greek sigma)
- SD : easier to understand because it's in the same units as your measurement.

- SD is a unique property of the normal distribution -- given a mean and a SD you have uniquely specified the distribution

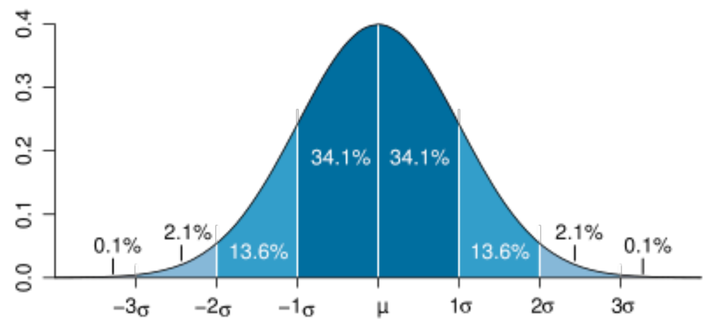
Standard Deviation

- SD = Square root of Variance

$$S = \sqrt{\frac{\sum_{i=1}^{N} (d_i)^2}{N}}$$

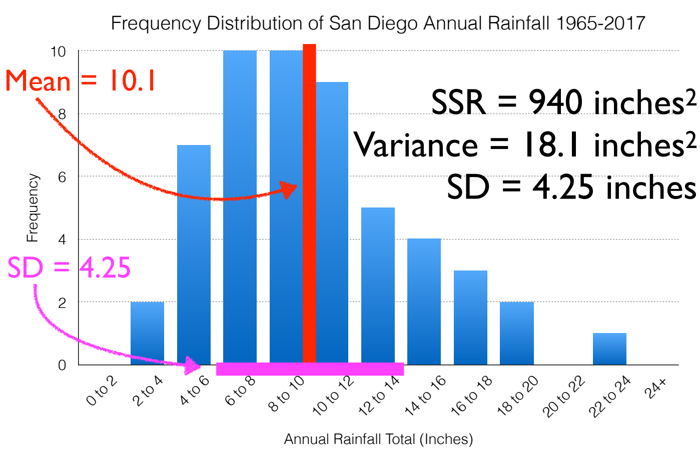
$$S = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}}$$

Normal Distribution



In a normal distribution, about 68.2% of values fall within ±1 SD

SSR, Variance and SD



Central Limit Theorem

- Basic gist : no matter what the Population distribution looks like, if you take enough (*) samples of the mean, the distribution of your samples of the mean will have a Normal distribution
- Central Limit Theorem Exercise (Javascript)
- This fact makes our life easy: Many statistics assume a normal distribution. The CLT provides us a normal distribution in most cases, even when the population data is skewed

Exercise: normal distribution

- Roll one 10-sided die 10 times and record the results
- Prediction
  - Your Distribution: Uniform (flat)
  - Mean : 5.0
  - Class Distribution: ???
- hint: What is N? # die rolls, # of students?
- List and Graph results
- Does the distribution look normal?
  - if so, why?

Exercise 1: Die Rolls

X	M	d = (x_i - M)	(residual)²
1	3	-2	4

N	M = $\bar{X}$	Σ Residuals	Σ (residual²)	$\frac{\Sigma\ (\text{residual}^2)}{N-1}$	$\sqrt{S^2}$
	3				

Histogram

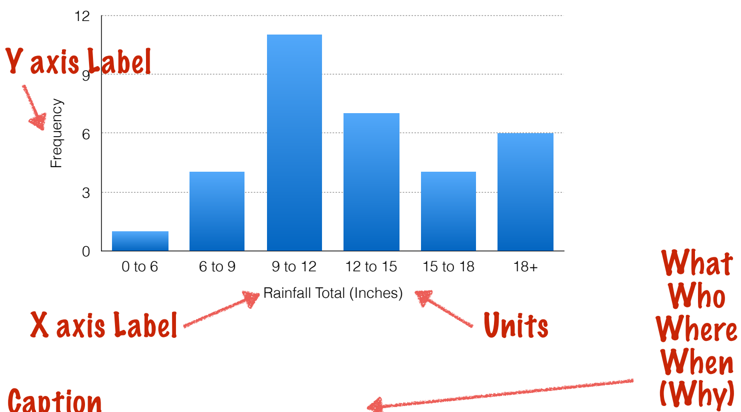


Figure. 1: Frequency Distribution of San Diego Rainfall 1970-2002

Exercise: normal distribution 2

- Compute Mean ($\bar{X}$) - is it near 5.0?
- Compute residuals
- Compute sum of residuals -- do they add to zero?
- Compute squared residuals
- Compute Sum of squared residuals (SSR)
- Divide SSR by (N-1) - this is Variance or (S^2)
- Take square root of variance - this is S or Standard Deviation
- For this exercise, SD should be near 2.8

Review

- Why Numbers?
  - pros
  - cons
- Distributions
  - Tables
  - Graphs

Review

- Frequency Distribution aka Histogram
  - graphically shows data
- Central Tendency
  - mean, median, mode
- Dispersion or Variation
  - residual
  - sum of residuals = 0
  - sum of squared residuals > 0
  - SSR/N = Variance
  - Sqrt(Variance) = Standard Deviation

Exercise: key points

- Some events (such as the roll of a die) have a flat (or 'uniform') distribution, but these are rare.
- Many big events are composed of many small events.
- Events in the real world often are distributed in a (nearly) normal distribution
- Assuming a normal distribution, the easiest way to describe the data is by two factors: Mean and SD.

Types of Statistics

- Descriptive:
  - Goal: help you describe the data
  - reduce the amount of data necessary for understanding
  - don't draw conclusions -- "just the facts"
- Inferential:
  - Goal: draw conclusions from your sample to the larger data set (population)

Measurement Scales

- Nominal
- Ordinal
- Interval
- Ratio

Nominal Scale

- Nominal: Name or ID only
  - red, blue, green....
    john, tony, fred...
    Sci2-243, Sci2-245...
  - does not signify Ordering, Ranking, or More/Less
  - Gotcha: even if used with Numbers it may be still a Nominal.
  - Example: colors, names, room numbers, ID numbers

Ordinal Scale

- Ordinal : ordering
  - first, second, third....
    1, 2, 3...
    A, B, C...
  - signifies Order, but can't assume distance between items is the same, e.g. the difference between an A and a B may be much different than a B and a C
  - Example: Class Rank, Assignment Grade, Product Ratings

Interval Scale

- Interval: specifies orders AND inter-item distance
  - -3, -2, -1, 0, 1, 2, 3.... 100, 105, 115
  - the difference between two numbers IS the same, e.g. 100 to 105 should be the same amount as 105 to 110
  - Does NOT have an absolute zero.
  - Example: temperature in Degrees Farenheit

Ratio Scale

- Ratio: specifies orders AND inter-item distance and has absolute zero
  - 0, 1, 2, 3.... 100, 105, 115
  - the difference between two numbers IS the same, e.g. 100 to 105 should be the same amount as 105 to 110
  - Does have an absolute zero.
  - Example: temperature in Degrees Kelvin

Measurement Scales

	Magnitude	Equal Intervals	Absolute Zero
Nominal			
Ordinal	✓		
Interval	✓	✓	
Ratio	✓	✓	✓

Scales: Practical Info

- Nominal Scale: common
  - common stats: Count, Frequency, Mode
- Ordinal Scale: less common
  - stats: specialized "nonparametric" techniques required
- Ratio and Interval: common
  - Often can be treated identically with same statistical techniques

Descriptive Statistics

- Count (N)
- Range (minimum, maximum)
- Frequency Distribution (histogram)
- Rank order, percentile (%ile)
- Central Tendency
  - Mean
  - Median
  - Mode
- Variation / Dispersion (Variance, Standard Deviation)

Population vs. Sample

- Ideally, measure *everyone* to get the exact value (*Population parameter)*
- Practically, this is impossible.
- Take samples instead, and calculate the *Sample statistic*.
- The "Law of Large Numbers", "Sampling Theory", "Central Limit Theorem" makes life easier
- Central Limit Theorem Exercise (Javascript)
- Some formulas differ for *Population* vs. *Sample* (divide by N or divide by N-1 ?)

Population v. Sample

	Population	**Sample**
Definition	the entire set of items	the actual subset you measured
Descriptives	"Parameters"	"Statistics"
Symbols	Greek	Roman
Mean	μ	$\bar{x}$
Std. Deviation	σ	S
Variance	σ^2	S^2
Divide by	N	N-1

Law of Large Numbers

- If you take enough* samples, the sample mean approaches the population mean.
- Example: a coin has two sides. If heads=1 and tails = 0, then the average expected result is exactly 50% Heads (0.5) in the long run.
- However, if you flip a coin just a few times, getting exactly 0.5 is not likely.
- The LLN states that you will if you take enough samples.

* what is "enough"? Rule of thumb : 100.

LLN Demonstration

- Law of Large Numbers
- Demonstration with Coin Flips

Logical Fallacies

Logical Arguments 1

- Logical arguments or inferences generally have several components:
  - Premises
  - Conclusions
- Example:
  - Premise: All English people are musicians
  - Premise: John Lennon was English
  - Conclusion: John Lennon was a musician

Logical Arguments 2

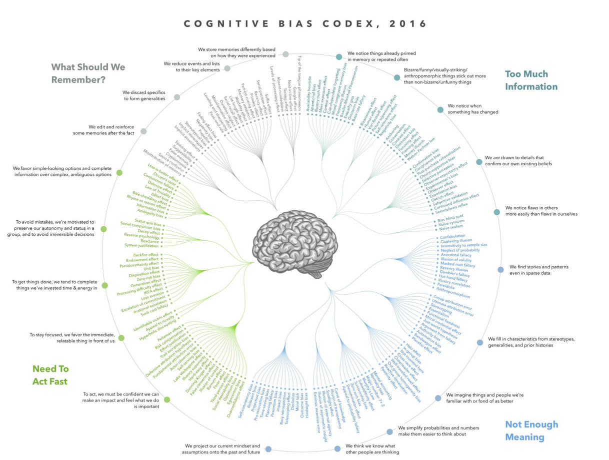
- An Inference can be either Valid or Invalid -- this refers to the Structure of the argument (not the Facts themselves)
  - All A are B
<u>All C are A</u>
All C are B
- A Valid inference can still come to a false conclusion, and vice-versa

Logical Fallacies

- A Logical Fallacy generally means that your inference is Invalid to begin with. In addition, your facts may or may not be true, but the flaw in reasoning has occurred before you even apply facts.

- Example: Affirming the consequent
- If P, then Q — bank owners are rich
Q is true — Bill Gates is rich
Therefore P — Bill Gates works at a bank

Cognitive Bias Codex



Nominal and Numeric

- <u>Nominal Fallacy</u>: The tendency to believe that something has a name or identification, it exists or has special meaning.
“I am sleepy” vs. “I am suffering from activity-induced-rest-reduction-performance-impairment syndrome”
- <u>Numerical Fallacy</u>: belief that something has been measured and assigned a number, it actually exists. “I’m really sad” vs. “I scored a 32 on the Beck Depression Inventory”

Biased Sample

- Every individual x that we have seen from sample X has characteristic Z
  Therefore ALL X have characteristic Z
- Every student I talk to in this class is interested in Psychology
  Therefore, ALL students are interested in Psychology

Reification Fallacy

- To Reify - to make something more concrete or real
- Examples:
  - "An A student"
  - "High IQ"
  - "Top of the class"
  - "An F Grade"

Ranking Fallacy

- Reducing a complex phenomenon (e.g. intelligence), giving it a single number (reification) and then ordering based on that number
- Examples:
  - A IQ of 93 is better than an IQ of 90
  - An income of $50,000 is better than $45,000

Other Fallacies

- Begging the question -- circular argument
- Correlation implies causation
- Post hoc ergo propter hoc (*after* this, therefore *because* of this)
- Appeal to Authority
- Ad-hominem
- Straw Man
- False Dilemma

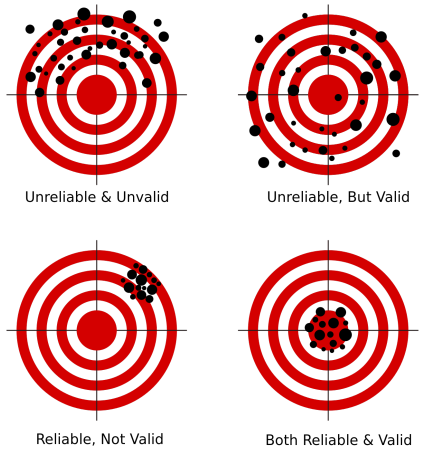
Precision vs. Accuracy

- Precision : the level of detail a measurement is made with, often specified with an error-range
  - "about 6 feet plus or minus 1 foot" vs. "6 foot 11 inches plus or minus 1 inch"
- Accuracy: how close the measured value is to the actual value, does it "hit the target"
  - Think arrow vs. shotgun
- A number can be precise and accurate, precise but inaccurate, or accurate but imprecise.

Precision Fallacy

- A number that is *precise* may seem to be *accurate* when it is not
- A measurement that is *reliable* may seem to have *validity* when it does not

Precision vs. Accuracy



- Shotgun vs. rifle analogy
- Similar to Reliability vs. Validity

Reasoning re: Probability

- Classical
- Gambler’s Fallacy
- Bayesian Reasoning

9 Heads in a row

- You are flipping a coin, and get 9 heads in a row
  H H H H H H H H H
- What is the % chance the next flip will be a H ?
- Three plausible answers:
  - 50/50
  - more likely Heads
  - more likely Tails

9 Heads: Classical Inference

- Coin flips are independent 50/50 events, therefore 50% : Logical/Statistical
- This is the *correct* answer for a fair coin

9 Heads: Gambler’s Fallacy

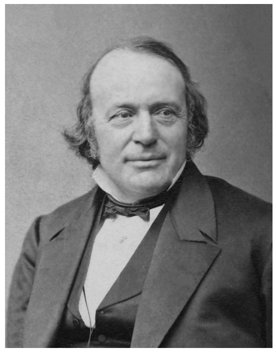
- Coin flips are independent 50/50 events, but I’ve seen 9/10 heads, therefore a Tail is “due”
- This is the “Gambler’s Fallacy” and the reason Casinos make tons of money. The reasoning is false.
- Note: when dealing with draws w/o replacement, this logic is *correct*. For example, a single-card blackjack deck -- if no face cards have come up after 30 cards, then face cards are due

9 Heads: Bayesian Statistics

- Coin flips are supposed to be 50/50 events, but I’ve seen 9/10 heads, therefore the data is telling us that perhaps this is not a fair coin.
- Bayes’ theorem suggests you evaluate the prior probabilities in determining future behavior
- In this case, you’d conclude that Head is <u>more</u> likely

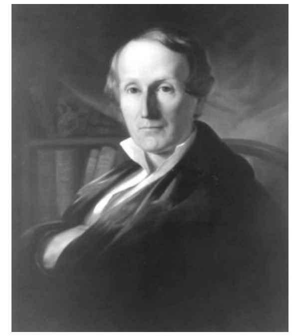
Louis Agassiz

- Swiss-born, European-trained biologist / geologist
- Came to Harvard in 1847
- Creationist -> Polygenist
- Taxonomist
- Resisted Darwin's theory of Evolution
- d. 1873

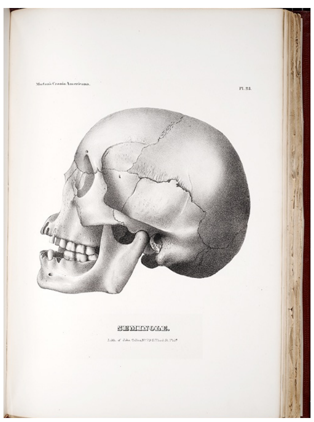


Review: Samuel George Morton

- Theory of Polygenism
  - Humans are composed of different species, created by god
- Craniometry
- Biological Determinism
- "Scientific Racism"
- The "American School"
- d. 1851



Crania Americana



Samuel George Morton
1839

Morton's Data as printed

Race	N	Cranial Volume Mean
Caucasian	52	87
Mongolian	10	83
American	144	82
Malay	18	81
Ethiopian	29	78

Data, corrected

Race	Mean (Morton)	Mean (corrected)
Caucasian	87	87
Mongolian	83	87
American	82	86
Malay	81	85
Ethiopian	78	83

Seed vs. Shot

Race	Difference (seed - shot)
Caucasian	1.8
Mongolian	n/a
American	2.2
Malay	n/a
Ethiopian	5.4

Morton's errors

- Fundamental arithmetic errors
- Data selection errors
- Failure to measure or control for external variables (gender, stature, etc.)
- Basic Statistical errors (averaging measurements from unequal size subgroups)
- The racist thumb press?
- Is he a liar? Conscious or subconscious?

Is skull size related to IQ?

- Skull size is related to IQ, but the relation is not terribly strong
- R is perhaps 0.20 to 0.40
  - What is R^2 ?
- Thus, the observed 3-4 cubic inch difference between the races would account for, *at most*, a 2-3 point IQ difference*

* Measured IQ differences between ethnic groups are actually much higher than 2-3 points -- reasons discussed later.

* 3 IQ points = about the Flynn effect per decade

Review

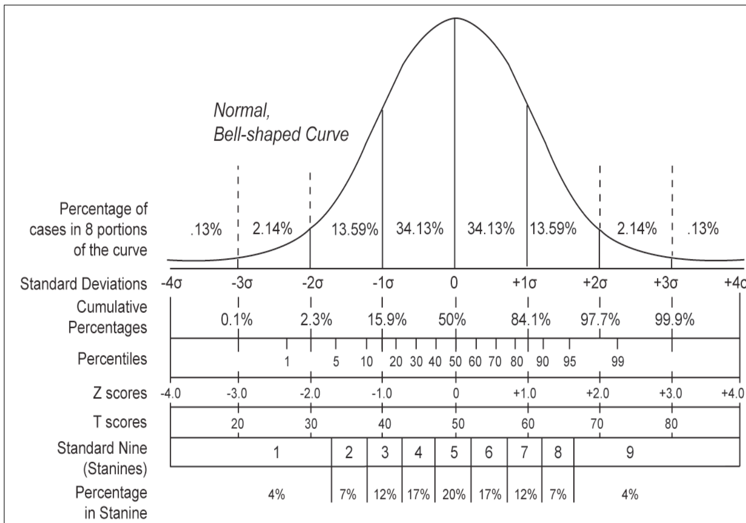
- Frequency Distribution aka Histogram
- Normal Curve
  - CLT, Mean (SD)
- Law of Large Numbers
- Scales of Measurement
- Population vs. Sample
- Logical Fallacies
  - Precision vs. Accuracy
    - Reliability vs. Validity
  - Gambler's Fallacy

Standard Scores

- Describe a single score in comparison to the population
- Ranks & Percentiles: useful
- Another way: difference scores
- Problem: Is a difference of "3" big or little? On a 100 point test it's not very large, but on a 10 point test it's the difference between an A and a C
- Want a system independent of the raw score units (just like letter grades)

Standard Scores 2

- Use the mean and standard deviation as points of reference.
- Standard score : distance from the mean, scaled by standard deviation
- Not affected by raw score units.
- Different standard scores mean the same thing, but are expressed differntly.
  - just like how 1.0 and 100% mean the same thing
- Unfortunately, there are several different Standard Score systems!



Standard Scores

	Z scores	IQ scores	T scores	Scaled Scores
Mean	0	100	50	10
SD	1	15	10	3
Example: top 3%				
Example: top 1%				
Formula to convert from Z Score	Z	(Z*15)+100	(Z*10)+50	(Z*3)+10

z-Score $(X-\bar{X})/s$	T-Score 10z + 50	Wechsler IQ (15z + 100)	Stanford IQ (16z + 100)	Scaled Score (3z + 10)	Percentile Rank
3	80	145	148	19	99.9
2.9	79	144	146	--	99.8
2.8	78	142	145	--	99.7
2.7	77	141	143	18.1	99.6
2.6	76	139	142	--	99.5
2.5	75	138	140	--	99.4
2.4	74	136	138	--	99.2
2.3	73	135	137	17.2	98.9
2.2	72	133	135	--	98.6
2.1	71	132	134	--	98.2
2	70	130	132	16	97.7
1.9	69	129	130	--	97.1

Norms 1

- Standard Scores provide us with a way of describing how a particular score relates to others in the population.
- Describing how an individual score relates to the population, which we assume are “normal”.
- Terms “normative data” and “norms”

- Key questions: What is the normative group? What features or factors of the group may affect scores?

Norms 2

- “norm-referenced” tests vs. “criterion-referenced” tests.
  Example: an 85 year old in excellent shape could be in the top 5% of his class for firefighting ability, but this may still be a “failing” grade.
- Common factors that may matter:
- Gender, Age, Education, Ethnicity/Race, Language, Handedness, Height, Weight…