

Ch. 3: Correlation & Linear Regression

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Ch. 3: Correlation & Linear Regression

- Relationships between 2 variables
- Scatterplots
- Linear Regression
- Exercise 2
- Correlation
- Race / DNA

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Number of variables

- One variable, one dimension
- Number Line
- Frequency Distribution / Histogram
 - 2 dimensional graph of 1D data
- Difference Score
 - 1 dimension
 - 2 dimensions

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Bivariate relationships

- “is factor A related to factor B”?
- Methods of analysis...
 - Anecdotal / Clinical
 - Numerical : simple 2x2 analysis
 - Visually -- scatterplots
 - see relationships and problems w/data
 - can't test hypothesis
 - Statistically -- correlation & regression
 - hard to detect problems w/data
 - easy to test hypothesis

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Anecdotal / Clinical

- Many interesting findings began from non-scientific approaches
- “Intuition” that something is related through experiencing multiple situations
- Pattern recognition - Good and Bad
- Problems -- faulty memory, confirmation biases, prejudice, etc...
- Next step after a “gut” feeling : design experiment and collect data.

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Simple numerical analysis

- Simplify:
 - use categorical variables
 - or convert continuous variables to categorical
- Use extreme cases to maximize effect
- Compute percentages in a 2x2 matrix
- Do the results suggest an effect?
- Compute Chi-square statistic to judge significance

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Dichotomous Variables

- The simplest form of categorical
- Aka “binary”
- Examples:
 - 1/0
 - yes/no
 - pass/fail
 - true/false
 - healthy/sick
 - normal/impaired
 - etc.

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Example

- “I think there is brain dysfunction in HIV disease” as measured by neuropsychological testing
- Medical status: control vs. HIV+ symptomatic
- NP test results: normal vs. impaired

		Medical Status	
		Control	HIV+
NP Status	Normal	85%	52%
	Impaired	15%	48%

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2x2 Analysis

- Pro: easy to understand
- Con: using binary categories reduces *statistical power*
- Conclusion: other Graphical and Statistical methods should be used as well.

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Scatterplots

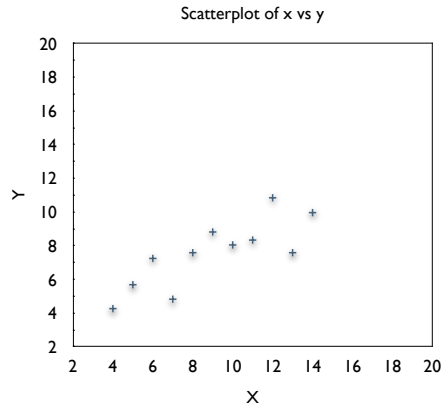
- Graph two variables in relation to each other on two-dimensional X, Y axis
- Easy to see
 - relations
 - problems
- Can't prove relationship is “significant”
- Difficult to interpret clinically or in “common sense” terms

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Scatterplots

x	y
10	8.04
8	7.58
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68



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Linear Regression

- Assume X and Y are related
- Assume relationship is linear
- Model with single straight line
- Pick the line that best “fits” our data
- Other names: fitting a line, finding the trend, creating a trendline, best fit line...
- Residuals = difference between prediction and actual value
- Linear Regression minimizes the square of the residuals, often called “Ordinary Least Squares”

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Why “Regression”

- Frances Galton
- Height of children vs parents.
- Tall parents have tall children (and vice versa)
- But children are closer to the mean than their parents (by a factor of ~2/3)
- Galton called this “Regression to the Mean”
- His paper fit** straight lines to data points.
- The technique has been called “regression” ever since
- ** He never calculated the lines, he just eyeballed them

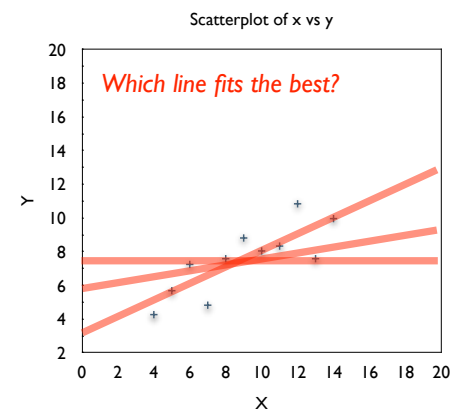
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Linear Regression

Equation:
 $y = 3.0 + 0.5x$

Correlation
 $r_{x,y} = 0.816$

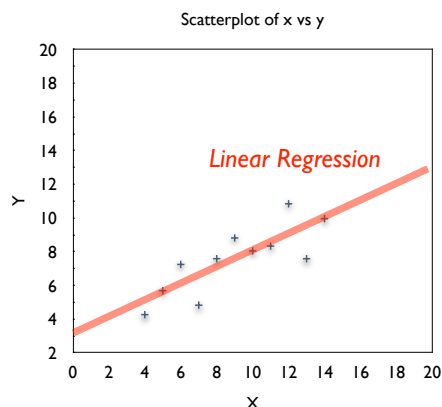


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Anscombe's Quartet I

x	y
10	8.04
8	7.58
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68

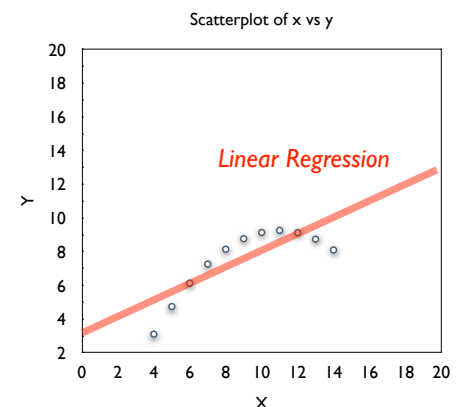


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Anscombe's Quartet II

x	y
10	9.14
8	8.14
13	8.74
9	8.77
11	9.26
14	8.1
6	6.13
4	3.1
12	9.13
7	7.26
5	4.74

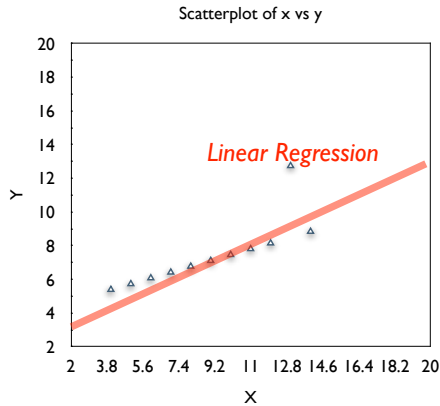


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Anscombe's Quartet III

x	y
10	7.46
8	6.77
13	12.74
9	7.11
11	7.81
14	8.84
6	6.08
4	5.39
12	8.15
7	6.42
5	5.73

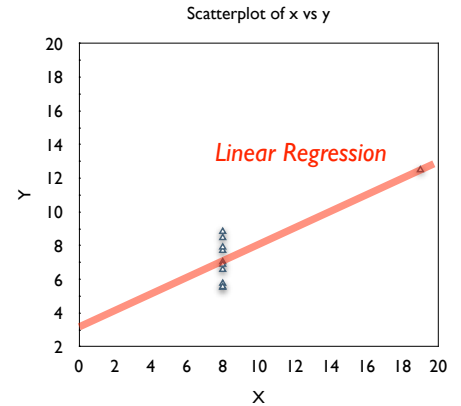


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Anscombe's Quartet IV

x	y
8	6.58
8	5.76
8	7.71
8	8.84
8	8.47
8	7.04
8	5.52
19	12.5
8	5.56
8	7.91
8	6.89

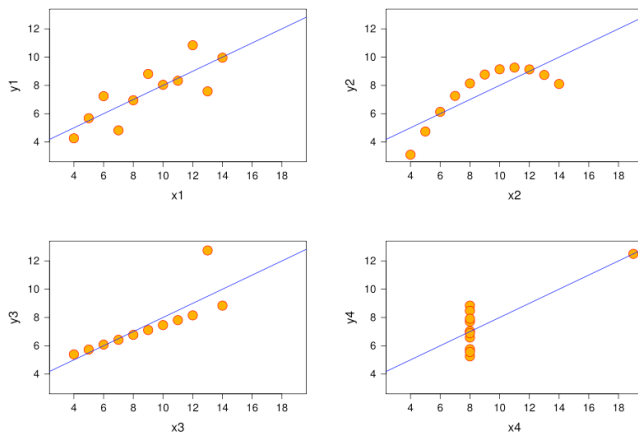


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Anscombe's Quartet

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Anscombe's Quartet Summary

- Each series has the same Quantitative stats:
 - linear regression equations
 - correlations
- Each one is Qualitatively different
- Each series needs special handling
- Lesson? Graph Your Data!

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Linear Regression Equation

$$Y' = a + bX$$

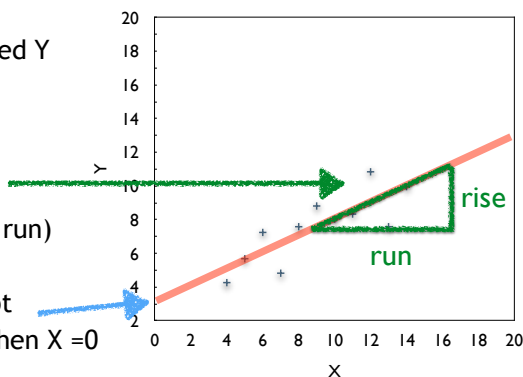
Y' = predicted Y

X = actual X

b = slope
 dY/dX
 (rise over run)

a = intercept
 Y value when X = 0

Scatterplot of x vs y



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Residuals in Linear Regression

- X : independent variable
- Y : dependent variable
- Model: predict Y from X
- Y' : "Y prime" : predicted Y
- $Y' = a + bX$
- Prediction is imperfect.
- Difference between predicted (Y') and actual (Y) is called a "Residual" = $(Y - Y')$
- Calculation of best fit line minimizes the sum of the squared residuals $\sum (Y - Y')^2$

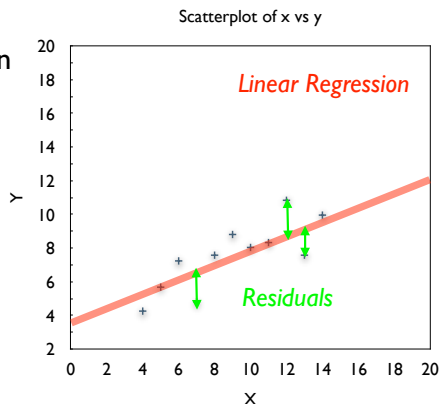
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Residuals in Linear Regression

Residuals are difference between actual Y and predicted Y' ($Y - Y'$)

Graphically it is equal to how far away (vertically) a point is from the linear regression line

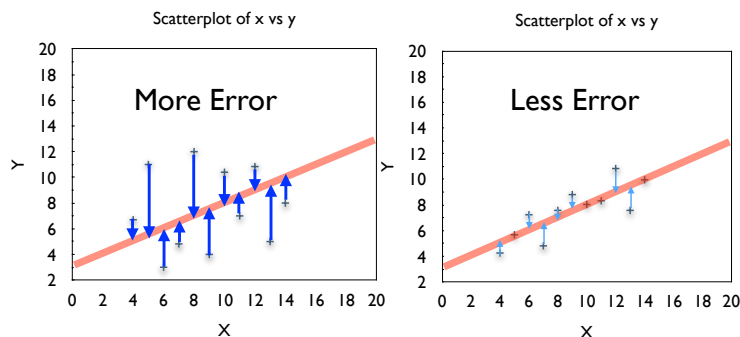


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Residuals and Error

Residuals (error) are greater when Y values are further from prediction.



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Residuals

$$d_i = y_i - y_i'$$

- In linear regression, the difference between the predicted y and actual y

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Measuring "fit"

- Can we use residuals to measure how well the predicted values measure the actual values?
- E.g. how big are the residuals
- *Similar to how we calculate Standard Deviation with a single X variable*

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Sum of Squared Residuals

$$SSR = \sum_{i=1}^N d_i^2$$

$$SSR = \sum_{i=1}^N (y_i - y_i')^2$$

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Sum of Squared Residuals

- Residual = $(Y_i - Y_i')$
- Squared residual = $(Y - Y')^2$
- SSR: Sum of squared residuals
 - Linear regression minimizes this value
- SSR is hard to interpret
- Can we standardize SSR?
- Need to compare SSR to something else

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Sum of Squares Total

- What can we compare SSR to?
- SST
 - similar to the null hypothesis:
 - “what would SSR be if X and Y aren’t related at all?”
 - uses the mean of Y as the model

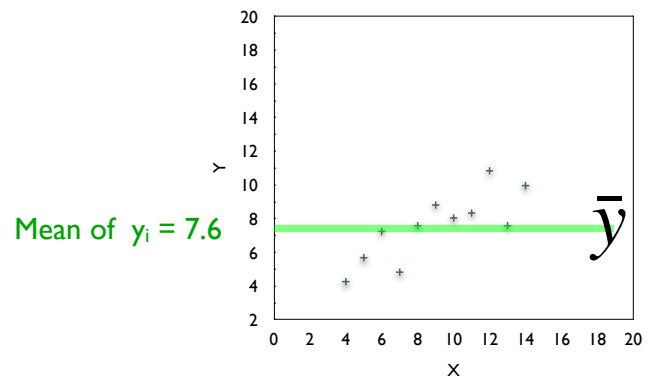
$$SST = \sum_{i=1}^N (y_i - \bar{y})^2$$

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$$SST = \sum_{i=1}^N (y_i - \bar{y})^2$$

Scatterplot of x vs y



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R^2

$$R^2 = 1 - \frac{SSR}{SST}$$

- $R^2 = 1 - (SSR/SST)$
- Ranges from 0 to 1 (0% to 100%)

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R^2

- Terminology
 - Coefficient of Determination
 - Explained Variance
 - Shared Variance
- Meaning
 - what % of variation in Y values can we predict from the variation in X values
- Careful: *Correlation* is not causation

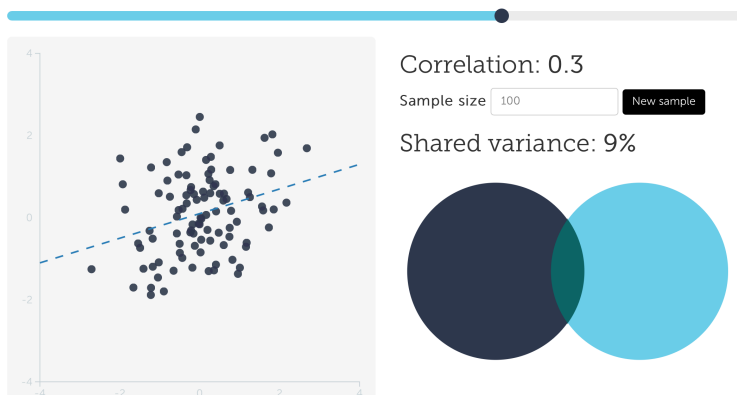
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Interactive Correlation Demo

- <http://rpsychologist.com/d3/correlation/>

Slide me



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Ch. 3 - Part 2

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Review

- # of variables / dimensions
 - 1 Mean (SD)
 - 2 Linear Regression

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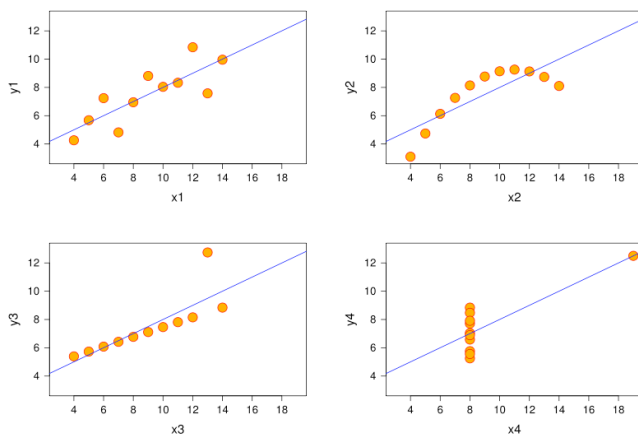
Review

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Anscombe's Quartet

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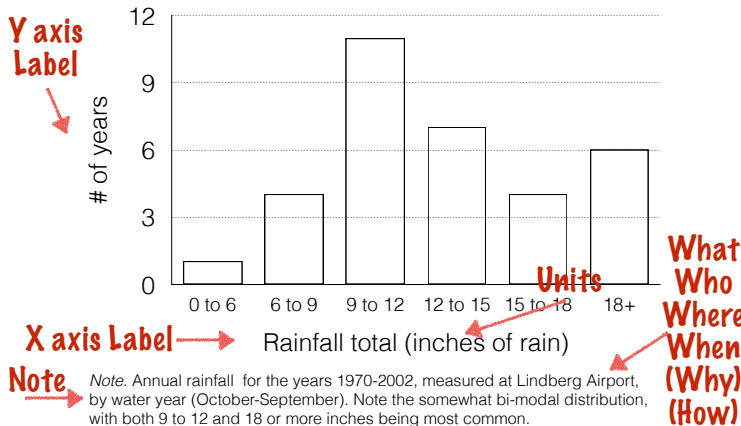
Exercise 2 - GraphPad Prism

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Example of APA-7 style Histogram

Figure 1
Frequency Distribution of Annual Rainfall in San Diego



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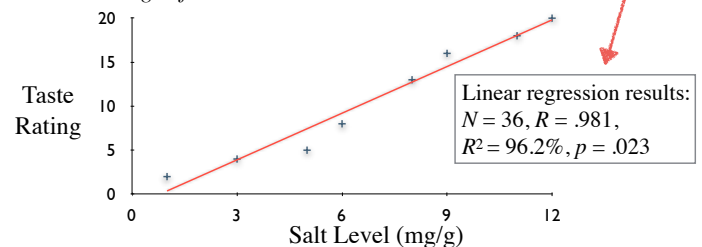
APA-7 Figure Example

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Title is above the figure

Legend is within the figure

Figure 1
Taste Ratings of a Cracker in Relation to Salt Amount



Note. Subjects ($N=36$) ate a single dry cracker which varied in the amount of salt (milligrams per gram) and rated the taste on a 20 point scale. Note the very strong correlation, suggesting higher salt levels are strongly related to taste ratings.

Note is below the figure

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Ch. 3 - Part 3

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Review

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Standard Error of Estimate

- Residual = $(Y - Y')$
- Standard Deviation of residuals
 - measure of “average” error
 - aka “Standard Error of Estimate”
 - In Prism: $S_{y,x}$

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Correlation : Pearson's r

- Pearson's Product-Moment Correlation
- Measures the strength of the linear relationship between two variables
- Ranges between -1.0 and +1.0
- Is a special case of linear regression, when both X and Y have been turned into Z scores.
- r is **transitive commutative** (correlation between X and Y is same as correlation between Y and X)
- R^2 = “explained variance” is the proportion of variation in the data explained by the model.
- R^2 ranges from 0 to 1.0 (0% to 100%)

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Regression vs. Correlation

	Linear Regression	Correlation
Scores	Raw	Z
Mean, Std Dev	sample means sample Std Dev	0 1
Equation	$Y' = a + bX$	$Y' = rX$
Slope	b = change in Y per change in X	r = correlation coefficient
Slope²	meaningless	R^2 = % variance explained
Commutative ?	no	yes, $R_{xy} = R_{yx}$

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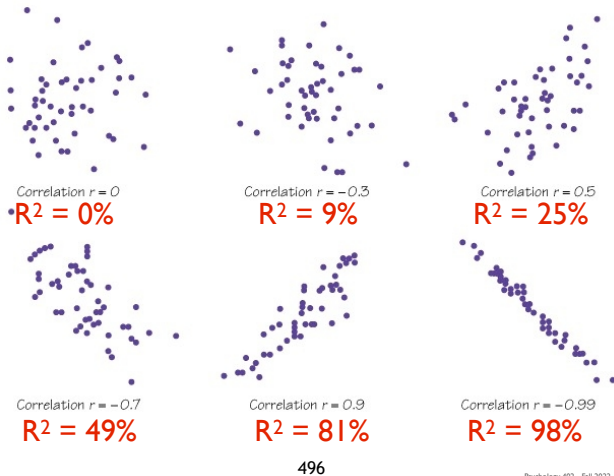
R vs R²

	R	R ²
Minimum	-1.0	0.0 (0%)
Maximum	1.0	1.0 (100%)
Meaning	correlation between X and Y	% of variance in Y explained by X
AKA	“correlation”, “correlation coefficient”	shared variance, explained variance, coefficient of determination
Notes	can be positive or negative	always positive (since it's squared)

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Correlations



Interactive Correlation Example

- <http://rpsychologist.com/d3/correlation/>
- R^2 or “Explained Variance” is sometimes called “Shared Variance”

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Other Correlation Coefficients

- Continuous (interval & ratio): Pearson’s r
- Ordinal (Ranked): A B C D... 1st, 2nd, 3rd...
 - Spearman’s Rho: correlation between two ordinal / ranked variables.
- Dichotomous (yes/no, one/zero, T/F, Male/Female, Pass/Fail...)
 - True vs. Artificial?

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Continuous vs. Dichotomous

Type of X / Type of Y	Continuous	Artificial Dichotomous	True Dichotomous
Continuous	Pearson r	Biserial r	Point biserial r
Artificial Dichotomous	Biserial r	Tetrachoric r	Phi
True Dichotomous	Point biserial r	Phi	Phi

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Correlation : Issues

- Technical / Calculation :
 - Non-normal distribution
 - Non-linear data and relationships
 - Outliers, data errors
 - Restricted Range
- Interpretation:
 - Correlation \neq Causation
 - Third variable explanations

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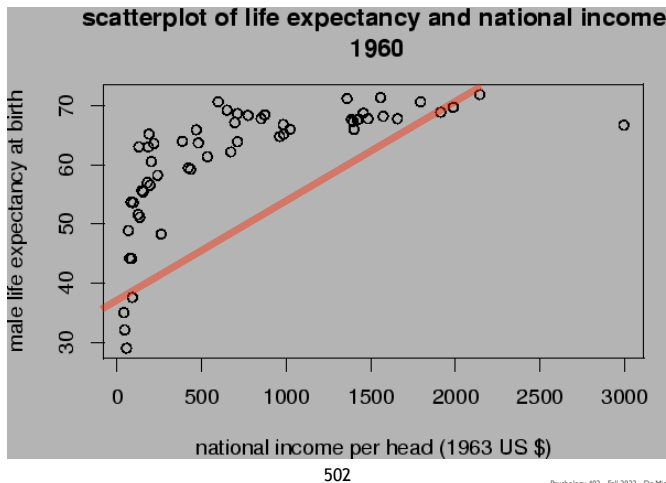
Non-linearity

- Linear Regression & Correlation assume a linear relationship between X and Y
- When it’s not linear:
 - Restrict the range of X
 - Transform (log, square root, etc.)
 - other statistical analyses (Spearman’s Rho...)

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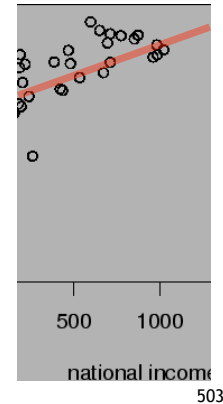
Life expectancy / national income



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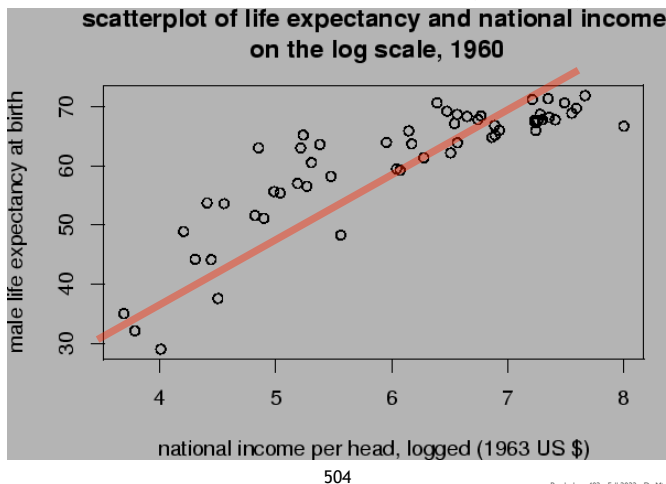
Restrict range of X



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log transform X (or Y)

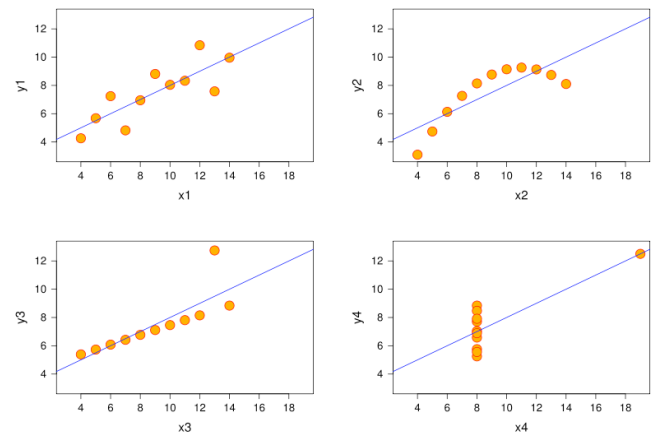


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Outliers & Data Errors?

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Correlation = Causation?

- A relationship (linear or otherwise) between X and Y tells us nothing about whether X causes Y
- Lack of correlation between X and Y does not mean that X doesn't cause Y
- Ice cream sales are positively related to increases in drowning deaths

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Hypothesis Testing

- Parameters estimated from sample data have error
- How do we know if a given estimate is correct?
- How big is the error likely to be (confidence intervals)?
- Inferential Statistics - covered later
 - Formulas to calculate probability, confidence intervals.
 - Higher N is better
 - “statistical significance” not the same as “clinical significance”

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Statistical vs Clinical Significance⁵⁰⁹

- Regarding the change in the Dependent Variable (DV)
- Statistical Significance:
 - Could the change be due to chance?
 - P value ($p < .05$: less than 5% probability)
- Clinical Significance
 - Was the change big enough to matter?
 - Effect Size (R^2)
 - Depends on context

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Significance vs. Effect Size

- Two coin flips : both heads (100%)
 - big effect size (50%)
 - not statistically significant ($p=0.25$)
- 1000 coin flips, 490 heads (49.0%)
 - small effect (1%)
 - statistically significant ($p=0.02$)
- 1000 coin flips, 350 heads (35%)
 - big effect (15%)
 - statistically significant ($p<.00000001$)

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Lies, damned lies, and statistics

- Statistical significance (P) is a function of...
 - Errors of measurement (E)
 - Effect Size (R)
 - Sample Size (N)
- $P \sim E / (R \times N)$

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Reporting Results

- Headline: “Men had higher IQ than women. Results were significant $p < .001$ ”
- → “that’s very significant”
- → “men are much smarter than women”
- P-value : statistically significant: Yes
- Effect Size : clinically significant: ? Unknown

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Review : Is race “real”?

- Pre-DNA theory
- Post-DNA theory

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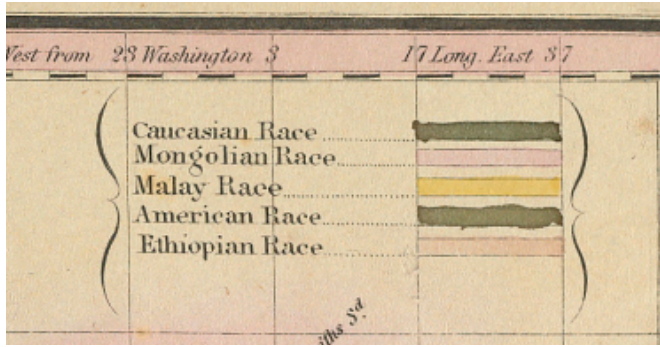
Pre-DNA

- Gold, Silver, Brass, Iron -- Plato
- “There is a physical difference between the white and black races which I believe will for ever forbid the two races living together on terms of social and political equality.” -- Abraham Lincoln

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Five Races?

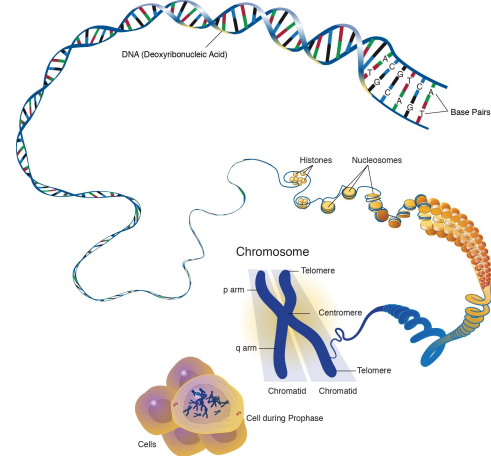


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Genetics : DNA

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Genetics

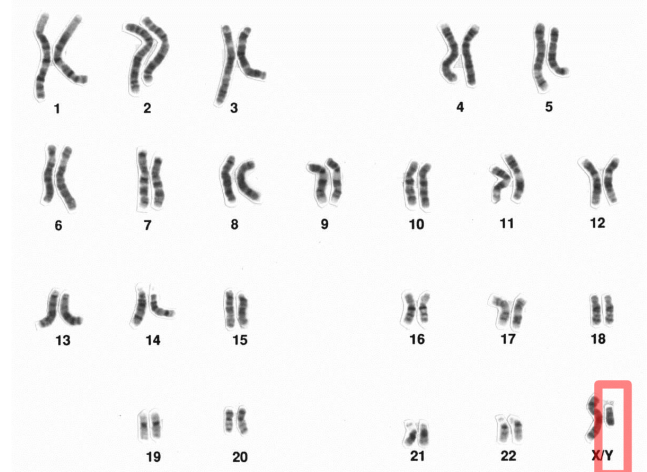
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- Human genome contains about 3 billion pairs of deoxyribonucleic acid (DNA)
- DNA is Transcribed into RNA
- RNA is Translated into Proteins
- Proteins
 - serve as structural components
 - function as enzymes to catalyze biochemical reactions
- Human DNA is grouped into 46 chromosomes
 - 23 pairs, one of each pair comes from each parent
 - 22 pairs in both males and females (autosomes)
 - 1 pair determines sex: either "XX" (females) or "XY" (males)

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Humans: 46 Chromosomes - 23 pairs

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Gene

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- DNA is subdivided into Chromosomes
- Chromosomes are subdivided into Genes
- Gene is a functional unit of DNA
- makes one thing (single protein or RNA)

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Genetics : Species Differences

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organism	estimated size (base pairs)	# genes	gene size	# chromosomes
Homo sapiens (human)	3.2 billion	~25,000	1 gene per 100,000 bases	46
Mus musculus (mouse)	2.6 billion	~25,000	1 gene per 100,000 bases	40
Drosophila melanogaster (fruit fly)	137 million	13,000	1 gene per 9,000 bases	8
Arabidopsis thaliana (plant)	100 million	25,000	1 gene per 4000 bases	10
Caenorhabditis elegans (roundworm)	97 million	19,000	1 gene per 5000 bases	12
Saccharomyces cerevisiae (yeast)	12.1 million	6000	1 gene per 2000 bases	32
Escherichia coli (bacteria)	4.6 million	3200	1 gene per 1400 bases	1
H. influenzae (bacteria)	1.8 million	1700	1 gene per 1000 bases	1

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Visible differences?

Indigenous
Australian
Melanesia
African
European

Australian and
Africans are
most genetically
different

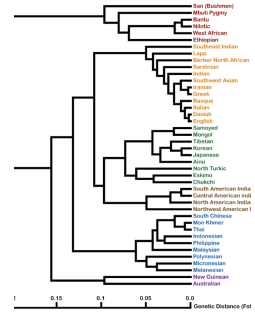


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Genetic Differences

- Sub-Saharan African
- Indo-European
- East Asian
- Native American
- South Asian
- Aboriginal



F_{st} = % of
subpopulation
variance

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DNA Variation

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DNA Variation

- variation between individuals : 3mbp / person
- variation within groups : 85%
- variation between groups: 15%
 - 5% - within *population groups*
 - 10% - between *population groups*
- *Note: skin color is one of the few traits where the pattern is reversed*

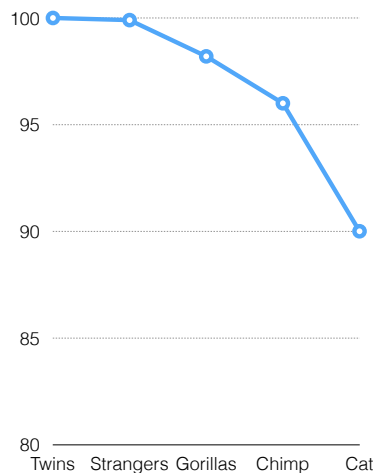
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DNA Differences

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- Identical Twins
 - 0.0%
- Human vs. Human
 - 0.1%
- Humans vs Gorillas
 - 1.6%
- Humans vs Chimps:
 - 4.0%
- Humans vs. Cats
 - 10.0%



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Post-DNA theory

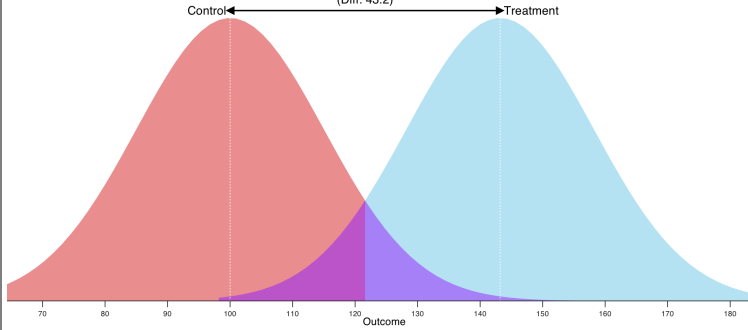
- Variance
 - variation between individuals
 - aka variation *within races population groups*
 - variation *between population groups*

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Skin Color

Cohen's d: 2.9
(Diff: 43.2)



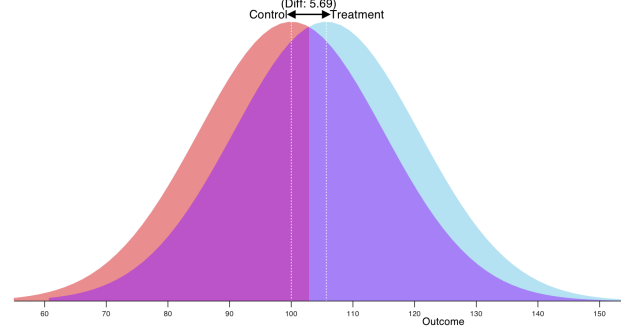
- 85% between group, 15% within group
- 98% probability blue person higher than red

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Many other traits

Cohen's d: 0.38
(Diff: 5.69)



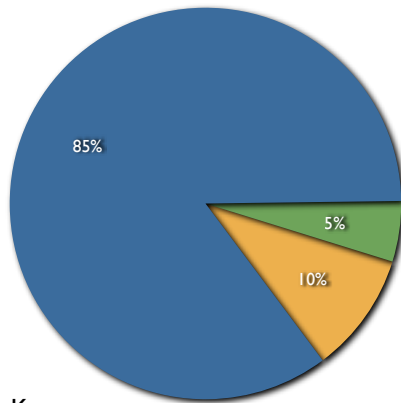
- 15% between group, 85% within group
- 61% chance blue person higher than red

532

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Variance: Genetic Variation

- Within local populations
- Within "race"
- Between "race"



For example:

- 85% within Japanese
- 5% between Japanese & Korean
- 10% between Asian and Caucasian

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Prehistorical Migration

