

Ch. 4: Reliability

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Reliability

- Constructs & Measurement
- History
- Classical Test Score Theory
- Four Kinds of Reliability
- Standard Error of Measurement
- Increasing Reliability

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Constructs & Measurement

- Psychology as “soft science”
- Construct
 - exists but can’t be directly measured
 - examples
- Measurement
 - “true value” - intelligence
 - measured or *observed* value (e.g. IQ test score)
 - discrepancy - “error”
 - How to conceptualize *error*?

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History 1

- 1896 - Karl Pearson - product-moment correlation (for continuous variables)
- 1904 - Charles Spearman - “*The proof and measurement of association between two things*” - *Rho* - correlation for Ordinal variables

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History

- Pearson, Spearman, Thorndike (1900-1907)
 - Basic reliability theory
- Kuder, Richardson (1937), Cronbach (1989)
 - Reliability coefficients
- Bartholomew & Knott (1990s)
 - Latent variable theory
- Drasgow et al (late 1990s)
 - Item Response Theory (IRT)

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Classical Test-Score Theory

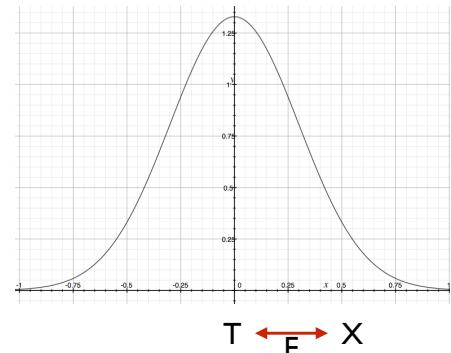
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- T = True Score
- X = Observed
- E = Error

- X = T + E
- E = X - T



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Classical Test-Score Theory

- True score (T) : the “actual” score that exists
- Observed score (X) : score as measured by a test
- Error (E) : difference between Observed and True score
- $X = T + E$
- $E = X - T$
- Assumptions: True scores have no variability. Errors are random (e.g. a normal distribution with mean of zero)

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Classical Test-Score Theory: Reliability

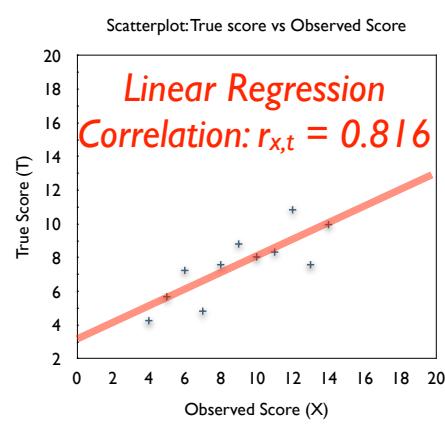
- Reliability = correlation between Observed score and True score
- $R_{X,T}$

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Classical Test Score Theory

X	T
10	8.04
8	7.58
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68



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Models of Reliability

- Most reliability measures are Correlation coefficients
- Alternate definition: Reliability is the ratio of the variance of True scores to the variance of the Observed scores
 - $\rho^2_{XT} = \frac{\sigma^2_T}{\sigma^2_X}$
- Or, it's the “Signal to Noise” ratio
 - $\rho^2_{XT} = \frac{\sigma^2_T}{\sigma^2_T + \sigma^2_E}$
- A test with reliability of $r^2=0.40$ means that 40% of variation in test scores is due to variation in the “true” score, and 60% of variation is random or chance factors.

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Sources of Error

- “Error” is considered the difference between True score and Observed score
- Where does Error arise?
 - Measurement errors
 - Change in True score
 - Sampling issues
 - Observer effects
 - etc...

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Measuring Reliability in Practice

- Since True score is hidden, can’t use the direct formula: $R_{X,T}$
- Instead
 - think about sources of error
 - practical methods
 - *estimate* reliability

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Test-Retest Reliability

- Test-Retest
 - administer test, delay for interval, administer test again
 - compute correlation between two administrations
 - same subjects, exact same test, two administrations

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Test-Retest Reliability

- Pros
 - easy for experimenter to do
- Cons
 - what causes error?
 - short testing interval → practice effects
 - long testing interval → change in true score over time
 - subjects have to take test 2x

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Parallel Forms Reliability

- Also called “alternate” or “equivalent” forms
- Item Sampling
- administer two versions of the test to same subjects (can have zero delay)
- compute correlation between two scores
- same subjects, different test forms, two administrations
- Pros: more rigorous method of determining reliability
- Cons: more work: experimenter must make a second test, subjects have to take 2 tests

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Internal Consistency Reliability

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Internal Consistency Reliability

- Give single test, calculate internal consistency of various subsets of items
- Only one test, one administration, same group of subjects
- Older methods:
 - Split half method
 - Spearman-Brown formula
 - KR20 formula
 - average of all possible Split Halves
 - can only handle right/wrong scoring
 - newer methods are better...

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Internal Consistency Reliability

- New: **Cronbach's Alpha (α)**
 - estimates a lower bound for reliability
 - does not require right/wrong scoring
 - can be used with Likert scales
 - $\alpha \geq .90$ is good
 - $\alpha \geq .80$ is ok
 - α between $.70 - .80$ is borderline
 - $\alpha < .70$ is bad

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Inter-Rater Reliability

- Observational data differs from self-report data.
- Even though most behavioral rating systems attempt to be precise, errors occur (e.g. did the child fall down? or were they pushed?)
- We must consider the reliability of different observers (also called "raters")
- **Cohen's Kappa**
 - ranges from -1 to +1
 - "poor" $< .40$
 - "good" $.40$ to $.75$
 - "excellent" $> .75$

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Reliability: errors & methods

	Description	Name	Statistic
Time Sampling	1 test given two times	test-retest reliability	correlation between scores at two times
Item Sampling	2 different tests given once	Alternate or Parallel forms	correlation between scores on 2 versions
Internal Consistency	One test, multiple items	Split Half or internal reliability	Cronbach's Alpha
Observer Differences	One test w/ 2+ observers	inter-observer reliability	Kappa

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Quiz: What kind of Reliability?

Procedure	Source of error?	Reliability?
Olympic judges giving consistent scores for a gymnastics performance		
Correlation between your IQ test score taken at age 12 and again at age 13		
Correlation between scores on 2 versions of the midterm (assuming each student takes both versions)		
Correlation between student scores on questions 1-25 vs 26-50 of the midterm.		

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Summary

- Reliability
 - how consistent measured scores are
- Error
 - $E = X - T$
- What kind of Error?
 - test-retest, item sampling, internal consistency, observer-differences

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Standard Error of Measurement

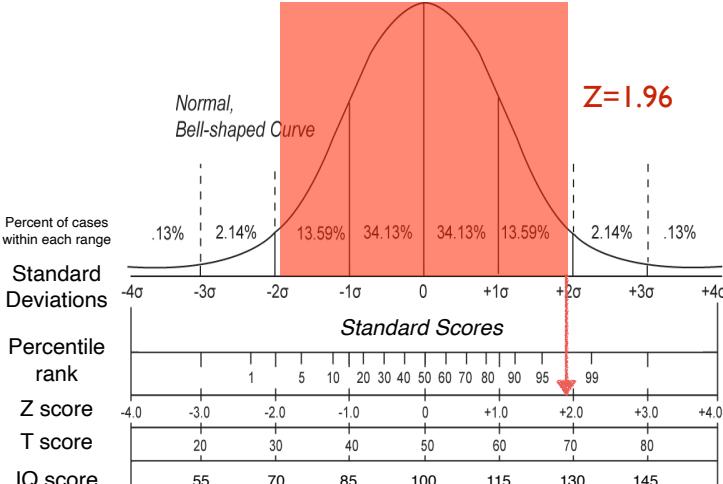
- Desire to answer question “how close is this test result to the true result”
- If we know the Reliability (r) of the test, we can estimate the likely range of true values
- Given
 - S = std dev of measured scores
 - r = reliability coefficient of test

$$SEM = S\sqrt{1 - r}$$

$$\sigma_{meas}$$

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SEM example: IQ

- Example: a person scored 106 on an IQ test, that has a reliability of 0.89. What is the 95% confidence interval of their true score
- $S = 14$
- $r = 0.89$
- $SEM = S\sqrt{1 - r}$
- $SEM = 14\sqrt{1 - 0.89}$
- $SEM = 4.64$
- Next, compute a confidence interval

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Confidence Interval

- “How likely is a true score to fall within a range”
- $Z = z\text{-score associated with \% range}$
- Confidence interval = $Z * SEM$
- Example:
 - 95% confidence interval : $Z = 1.96$
 - $SEM = 4.64$
 - $1.96 * 4.64 = 9.1$
 - 95% CI = ± 9.1 points
 - Range = $X \pm CI$
 - $106 \pm 9.1 = \text{range from } 96.9 \dots 115.1$

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SEM Exercise

- This is for practice, not scored for points

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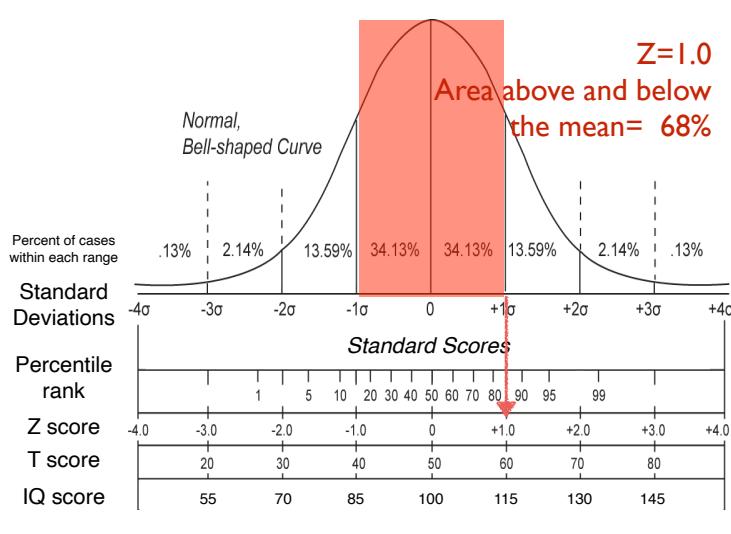
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Common Z scores & Confidence Levels

Z Score	Area above mean	Area above + below Mean	Proportion as %
0.00	0.000		0%
0.13	0.051		
0.67	0.249		
1.00	0.341	0.682	68%
1.64	0.449		
1.96	0.475		95%
2.57	0.495		

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How reliable?

- $r = .70$ or $.80$ or higher is often considered “good enough” for much research
- $r > .90$ is very good
 - may not be worth effort to go higher
- Some real-world tests have $r > 0.9$
 - example: modern IQ tests

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Increasing Reliability

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Increasing Reliability

- Increase N (number of questions, items or tests)...
- Focus on common characteristic...
- Other methods (covered later)
 - Use **Item Analysis** (“discriminability analysis”) to find items that best measure a single characteristic
 - Use **Factor Analysis** to determine sub-characteristics of a single test

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Increase N

- N = number of questions or items or tests
- Formula: increase N to increase reliability
- $N_d = r_d (1 - r_o) / r_o (1 - r_d)$
 N_d = new N (times old N)
 r_d = desired level of reliability
 r_o = observed level of reliability

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Increase N - Examples

- $N_d = r_d (1 - r_o) / r_o (1 - r_d)$
- Example:
 - 20-item CES-D has reliability of $.87$.
 - We need $r = 0.95$
 - $N_d = 2.82$
 - new N is $2.82 \times 20 = 56$ items

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Increase N - Examples

- $N_d = r_d (1 - r_o) / r_o (1 - r_d)$
- Your 40-item test has reliability of .50.
- You want .90.
- $N_d = 9.0$
- new N is $9 \times 40 = 360!$

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Focus Test

- Reliability increases as a test focuses on a single concept or characteristic ("construct")
- Trying to capture multiple concepts in a single test reduces reliability
- Methods:
 - Informal – remove items with poor face validity (chapter 5)
 - Statistical:
 - Discriminability Analysis (chapter 6)
 - Factor Analysis (chapter 13)

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Reliability Summary

- Measurement Error occurs in all fields -- Psychology focuses on it
- Kind of Reliability : *where* the error came from
- Improving Reliability: more items, focusing test, discriminability, factor analysis
- Reliability is useful: calculate SEM to get Confidence Intervals
- Reliability is not Validity: Reliable tests aren't automatically valid
- A reliable test *might* be valid

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